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Calculating the total vote where the district magnitude is greater than one: A test of some algorithms using British local election data

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SUMMARY *Electoral analysis using aggregate data relies on the availability of accurate voting statistics. One vital piece of information, often missing from official electoral returns, particularly British local government elections, is the total number of valid ballot papers. This figure is essential for the calculation of electoral turnout. When voters have a single vote and official information about the number of ballot papers issued is missing, a figure for the total vote can still be derived. However, local elections in Britain frequently use a system of multiple-member wards, where voters have as many votes as there are seats to be filled. In such cases, calculating the total vote and, hence, the turnout does present a real problem. It cannot be assumed that all voters will use their full quota of votes or that voters will cast a ballot in favour of a single party. This paper develops and tests di erent algorithms for calculating the total vote in such circumstances. We conclude that the accuracy of an algorithm is closely related to the structure of party competition. The findings of this paper have a number of important implications. First, the difficulties in calculating the turnout in multiple-member wards are identified. This will inform the debate about public participation in the local electoral process. Second, the method for deriving a figure for the total vote has an important bearing on a number of other statistics widely employed in electoral analysis.*

1 Introduction

Official General Election returns usually contain the information necessary for electoral analysis. The same cannot be said for local elections in Britain, however,

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where authorities are sometimes deficient in making complete data available—particularly data relating to the number of ballot papers issued at an election. Where single-member wards are used, the absence of information on the total number of ballot papers issued is not critical. In such cases, the total vote can be derived by summing the votes for every candidate. This figure is then used as the basis for calculating the electoral turnout and party vote shares.

Unfortunately, many local authorities in Britain employ a system of wards where district magnitude (the number of seats to be filled in a ward) is greater than one. In a sizeable proportion of such cases, we do not have information regarding the number of ballot papers issued. Specifically, the problem potentially affects election returns for 32 London boroughs and a total of 6000 ward election results, dating from reorganization in 1964. Also affected are district authorities in England and Wales, where, since reorganization in 1973, more than 24 000 election results do not contain information regarding the number of valid ballot papers issued. The current maximum district magnitude is five, although it has reached 12 in the past.

Another area of electoral analysis affected by this problem is those parliamentary contests in Britain which used two- or three-member constituencies. Although the practice of using multiple-member constituencies was ended in 1948 (Butler, 1963), problems remain in compiling accurate historical electoral data. Craig (1989) devised his own method for calculating the total vote but, as he himself noted, this made no allowance for voters who did not use their full quota of votes. Indeed, after 1868, voters in three-member constituencies were only permitted a maximum of two votes, thus making the calculation even more complicated.

The purpose of this paper is to address the problem of how best to calculate the total vote where the district magnitude is greater than one, and where the official return makes no mention of the number of ballot papers issued. This problem has a direct bearing on any subsequent calculation of electoral turnout and on how best to calculate party shares under such ballot conditions. After giving a brief description of the underlying problem in compiling electoral statistics for multiple-member wards and the circumstances which facilitated analysis, we describe and test a number of algorithms designed to estimate the total vote.

2 The underlying problem

Consider a constituency which returns m members to the legislature. If every voter has m votes and uses them all, then the number actually voting can easily be deduced by dividing the total number of votes cast by m. However, if any voters fail to use their full vote allocation, then this procedure will lead to an underestimate of the number voting.

Following the London borough elections in 1994, data on valid ballot papers in each ward were actually published (Minors & Grenham, 1994). Furnished with these, we devised different algorithms for calculating the total vote and compared these with the official figures in 414 three-member wards. The amount of error for each method provided a test of each algorithm's accuracy. One difficulty in relying on London electoral data alone, however, was that the pattern of party competition in those particular elections differs from that found elsewhere. In particular, parties are more likely to field full slates of candidates in London and intervention by independents is rarer than in the shire districts. To overcome this, we also specifically sought and obtained official data on ballot papers issued in a selection of 449 three-member wards in shire districts with elections in May 1995.

3 Algorithms for calculating the total vote

This paper considers a total of nine algorithms for estimating the total vote. The first five algorithms, summarized next, essentially base their methodology on the arithmetic mean, taking each party's mean vote as its contribution to the estimate of the total vote. The first of these methods, i.e. the Rallings and Thrasher (1993) algorithm (hereafter referred to as method A), based on the earlier and simpler algorithm of Curtice and Payne (1983), deals with all parties whether major or minor, but clusters the independent candidates, according to the number of vacant seats.

If one of the major parties (i.e. Conservative, Labour and Liberal Democrat) fields only one candidate, the vote of the best-placed candidate of each party, whether major or minor, is used in the estimation of the total vote. If a minor party fields an incomplete slate and that slate is less than that of the smallest major party, then the minor party vote is averaged on the basis of the size of its incomplete slate. For example, in a contest where there are two Green candidates and one ratepayer candidate, but the smallest major party slate is three, then the average of the two Green party candidates' vote and the vote of the single ratepayer would contribute to the estimate of the total vote.

Independent candidates are treated very differently. The problem with treating all independent candidates—who, by definition, have no political label—as separate and distinct is that this method gives an exaggerated benefit to independent candidates over party candidates, by counting their votes individually. Independents, therefore, are clustered according to the number of vacant seats, and the vote of the best-placed candidate in each cluster is used to calculate the independent contribution to the estimate of the total vote. Should six independent candidates contest a three-member ward, for example, then the algorithm takes the sum of the votes of the best-placed and the fourth candidate. If there are nine independents, then this will give three clusters and the sum of the votes of independent candidates one, four and seven would be used as the independent contribution. If the number of independent candidates is less than the number of vacant seats, however, then the average independent vote is taken.

Another algorithm, which also uses the mean vote, was devised by Curtice and Payne (1991). This procedure (method B) differs from method A in the way that minor parties are treated. In this case, the votes of the independent candidates are grouped together as one party and treated in the same way as any other minor party. If all parties in the contest put up a complete slate, then the individual party contribution to the total vote is taken as the mean vote for each party, regardless of whether the party is a major or minor one. If this is not the case, then the algorithm depends on whether the incomplete slate is put up by one of the major parties or by a minor party.

If the Conservative, Labour or Liberal Democrat parties put up an incomplete slate, then the size of the slate of the major party with the least number of candidates is used as the basis of the mean calculations. If it is a minor party that fields an incomplete slate, then the algorithm is more complex. The major parties are treated as before but the minor parties' votes are adjusted before they are added to the estimate of the total vote. Let us assume, for example, that the minor party with the incomplete slate fields only one candidate. First, the mean vote for each of the major parties is calculated and the ratio of the mean party vote for each major party to the vote of the best-placed major party candidate is obtained.

Next,the average of these three ratios is calculated and used to adjust the minor party vote (i.e. the minor party vote is multiplied by the average ratio). This gives the minor party contribution to the estimate of the total vote.

If the minor party puts up more than one candidate, then the method is adjusted to fit the minor party slate. When the minor party puts up two candidates in a ward with three or more members for example, then the calculations proceed as before, except that the average of the top two major party candidates is used in the adjustment ratio calculations; the average of the minor party vote is calculated and it is this figure that is adjusted to give the minor party contribution to the total vote. If the major and minor parties both field an incomplete slate, and the total number of the minor party candidates is not less than the minimum slate of the major parties, then no minor party vote adjustment is required. Hence, in this case, each party's contribution to the estimate of the total vote is its mean vote calculated using the size of the slate of the smallest major party. If the major and minor parties both field an incomplete slate, but the total number of the minor party candidates is less than the slate of the smallest major party, then an adjustment is required and the algorithm proceeds as described previously.

The next three algorithms, also based on a simple arithmetic mean, are much easier to implement. The first algorithm simply uses the mean vote for each party—regardless of the size of the slate of the smallest major party—as its contribution to total vote. The independent candidates are grouped together and treated as a separate minor party. This is method C. Method D proceeds as method C but uses the smallest major party slate as the basis for the mean vote calculations. If the size of the minor party slate is less than that of the smallest major party slate, then the mean is based on the respective minor party slate. Method E uses the mean party vote as in method C, i.e. regardless of the size of the smallest slate, but clusters the independent candidate vote, as in method A.

The next two algorithms (methods F and G) are not based on the mean vote but, instead, are based on the vote of the best-placed candidate. Method F simply uses the vote of the best-placed candidate for each party, including the independents. However, method G also incorporates method A's treatment of the independent candidates into the 'top vote' algorithm. As a consequence, the votes of the independent candidates are clustered according to the number of vacancies.

Of the final two algorithms, method H takes the opposite approach to that of methods F and G, and uses the vote of the lowest-placed candidate for each party, including the independents, as its contribution to the estimate of the total vote. Naturally, this cannot but provide an underestimate of the actual vote, but it is included here to provide some kind of benchmark against which the other methods might be assessed. Method I simply sums the votes of each of the candidates, regardless of party, and calculates the average number of votes per vacant seat. Each of these nine algorithms is summarized in Table 1.

The criterion used here to test the performance of the total vote algorithms is their 'fit'. On this basis, the estimation errors from the superior algorithms for the total vote should be as small as possible. One way to test the performance of each algorithm, therefore, is to examine how closely each algorithm tracks the official figures. Because positive and negative errors will, to at least some extent, cancel out, any quantitative measure used to evaluate the algorithms must eliminate this problem. A convenient measure of accuracy is provided by the root mean squared error (RMSE).

TABLE 1. A summary of the algorithms

Algorithm	Structure of contest	Major parties	Minor parties	Independents
A		Mean (min slate)	Mean (min slate)	Clusters
B	Complete slate (major and minor)	Mean (complete slate)	Mean (complete slate)	As minor
	Incomplete slate (major)	Mean (min incomplete major slate)	Mean (min incomplete major slate)	As minor
	Incomplete slate (minor)	Mean (major slate)	Mean plus adjustment	As minor
C		Mean (no. of party candidates)	Mean (no. of party candidates)	As minor
D		Mean (min slate)	Mean (min slate)	As minor
E		Mean (no. of party candidates)	Mean (no. of party candidates)	Clusters
F		Top vote	Top vote	As minor
G		Top vote	Top vote	Clusters
H		Smallest vote	Smallest vote	As minor
I	Votes for all candidates summed and divided by number of available seats			

4. Findings

The nine algorithms for estimating the total vote were calculated for each of the 414 London and 449 shire district three-member wards. This exercise was then repeated, but only for those 200 wards in London and 175 wards in the shires that involve just the three major parties. Finally, the algorithms were calculated again but only for those contests where each major party fielded a full slate of candidates, i.e. where nine candidates were on the ballot paper. There were 176 and 74 such cases respectively. Our purpose in dividing the wards in this way was to test how far the accuracy of the various algorithms was affected by the precise structure of party competition.

Because neither minor parties nor independents are involved in the analysis of wards that feature only the major parties, the number of unique algorithms is effectively reduced to five when three-party contests with varying sized slates are examined. The 'top vote' algorithms (methods F and G) will give identical results, as will methods A, B and D. This is because each of these methods uses the size of the smallest major party slate as the basis of their mean vote calculations. Similarly, methods C and E will give identical results, because both use the arithmetic mean vote, regardless of the size of the major party slate. When those three-party contests that involve a full major party slate are considered, the algorithms produce just three possible outcomes. Each of the methods based on the mean vote will give the same results, as will both of the methods based on the 'top vote'. Method A will be used in the tables as the mean vote algorithm that encompasses methods B, C, D, E and I, and method F (encompassing method G) will be used as the 'top vote' algorithm. Method H, based on the minimum vote, will give the third set of estimates. The results of the analysis are presented in Table 2.

If we first examine the results for the London boroughs, Table 2 clearly demonstrates that, for all three categories of ward, both 'top vote' methods F and G are clearly superior. Method F, i.e. the algorithm that uses the vote of the best-placed candidate for each party, regardless of political party, gives a slightly lower RMSE.

TABLE 2. Results

Method	RMSE		Percentage underestimated		Mean error	
	London	Shires	London	Shires	London	Shires
All three-member wards[a]						
A	234	309	80	53	145	− 67
B	236	318	82	58	156	− 40
C	236	274	84	65	159	− 11
D	270	314	92	54	222	− 68
E	232	301	85	62	157	− 33
F	185	325	58	30	10	− 154
G	196	315	59	31	15	− 146
H	356	326	95	78	301	124
I	297	302	100	100	273	253
Three-party contests with varying slates[b]						
A	258	192	93	60	223	1
C	260	163	98	75	235	48
F	147	181	82	35	91	− 80
H	394	248	100	90	368	168
I	267	286	99	99	263	230
Three-party contests with full slates[c]						
A	270	159	99	99	249	141
F	147	108	87	58	106	− 10
H	404	304	100	100	379	140

Notes: The estimation error is defined as the actual number of valid ballot papers minus the estimated total number of ballot papers, giving a positive error when the algorithm underestimates the total vote and a negative error when the algorithm overestimates. [a]London, $n = 414$; shires, $n = 449$. [b]London, $n = 200$; shires, $n = 175$. [c]London, $n = 176$; shires, $n = 74$.

The conclusion as to the best algorithm for estimating the total vote in the shire districts is not so clear-cut. The RMSEs for all three-member wards show that method C is the best estimator, but there is more variation and greater magnitude in the RMSEs than were found in London. In general, therefore, the algorithms do not estimate as well for the shire district wards and none is outstandingly better than any of its rivals.

Looking specifically at the three-party contests, however, the picture begins to change. If we examine those with varying party slates, then the RMSE indicates that method C is again the best estimator but method F is very close in magnitude. (This is the ‘top vote’ method that gave the superior estimates for the London wards). Taking the three-party contests with only full-party slates, the conclusion is unambiguous. Method F is quite definitely the better estimator of the total vote and, noticeably, the order of magnitude of the statistics is now actually less than those for the London wards.

Looking at the final four columns of Table 2, the most striking feature is that, for the London wards, the better methods underestimate, whereas, for the shires, the better methods tend towards overestimation. On average, all methods for London underestimate, whereas all methods with the exception of the two inferior ones overestimate for the shires.

5 Conclusions

The conclusions are, to some extent, clear-cut: if the ward, whether London borough or shire district, features a three-party contest without the intervention of minor parties or independent candidates, then the best estimator of the total vote is found to be the algorithm that uses the vote of the best-placed candidate for each party (method F). If the contest involves minor parties and/or independent candidates, then the empirical evidence is somewhat mixed. In the London boroughs, with their higher degree of party and candidate competition and lower level of intervention by independent candidates, the evidence for all three-member wards confirms the results from the three-party contests and points to the ‘top vote’ algorithm as providing the best estimation (method F). By contrast, the total vote in the sample of shire district three-member wards, with their lower level of party and candidate competition, including both incomplete major party slates and a much higher level of independent candidate intervention, is better estimated using a simple arithmetic mean (method C). Here, however, it must be stressed that none of the algorithms clearly stands out from the rest as being the best estimator.

In addition to conclusions about the optimum method for calculating the total vote where the district magnitude is greater than one, there are a number of other observations we should make. First, there appear to be significant differences in the patterns of voting, irrespective of the structure of party competition, between the urban London boroughs and the more suburban or rural shire districts. A number of factors, including the pattern of party organization and campaigning, as well as differing attitudes among urban and rural voters, were suggested as contributing to such differences. Further research is needed, which might focus on the range of votes between candidates of the same party, in order to discover the real extent of this phenomenon.

Second, our analysis has important implications for the calculation of the turnout in such cases. Overestimation of the total vote will, by definition, inflate estimates of electoral participation. Likewise, an underestimate of the actual number of voters will generate a lower figure for turnout. As the discussion of low rates of electoral participation grows, this evidence will have to be taken into account.

Third, this paper will have an important bearing on the way in which vote shares for individual parties are calculated. Clearly, there is no definitive solution to this problem, since the specific structure of party competition has a critical bearing on the vote share for each party. Once again, those engaged in any form of electoral analysis dependent on party vote share in multiple-member districts need to be aware of our findings.

There is no question that this paper has identified more problems than solutions. While the different algorithms have met with varying degrees of success, no single method achieved an acceptable level of accuracy. This could lead us to conclude that, when confronted with this problem, we should simply operate a policy of ‘horses for courses’, in so far as the structure of each electoral contest should determine which of the competing algorithms should be used. This might be a way forward but it is based on limited experience. After all, we have analyzed only two years of local electoral contests and these data might have different characteristics from those in previous and subsequent years. There is no doubt that multiple-member constituencies in single-ballot, simple plurality systems provide a fascinating source of data for aggregate analysis. It also appears that significant compromises are required to make the best use of that resource.